Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Remove logarithms and state $4-3^x=e^{1.2}$, or equivalent	**B1**	Accept $4-3^x=3.32(01169.....)$ 3 s.f. or better
	Use correct method to solve an equation of the form $3^x=a$, where $a>0$.	**M1**	$(3^x=0.67988..)$ Complete method to $x=\ldots$ If using $\log_3$ the subscript can be implied
	Obtain answer $x=-0.351$ only	**A1**	CAO must be to 3 d.p.
		3	

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Question	Answer	Marks	Guidance
2	Use correct quotient rule or correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2\mathrm{e}^{-2x}\left(1-x^2\right)+2x\mathrm{e}^{-2x}}{\left(1-x^2\right)^2}$
	Equate derivative to zero and obtain a 3 term quadratic in x	**M1**	
	Obtain a correct 3-term equation e.g. $2x^2+2x-2=0$ or $x^2+x=1$	**A1**	From correct work only
	Solve and obtain $x = 0.618$ only	**A1**	From correct work only
		5	

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Question	Answer	Marks	Guidance
3	Commence division and reach partial quotient $x^2 + kx$	M1	
	Obtain correct quotient $x^2 + 2x - 1$	A1	
	Set their linear remainder equal to $2x + 3$ and solve for a or for b	M1	Remainder = $(a+3)x + (b-1)$
	Obtain answer $a = -1$	A1	
	Obtain answer $b = 4$	A1	
	Alternative method for question 3		
	State $x^4 + 3x^3 + ax + b = (x^2 + x - 1)(x^2 + Ax + B) + 2x + 3$ and form and solve two equations in A and B	M1	e.g. $3 = 1 + A$ and $0 = -1 + A + B$
	Obtain $A = 2,\ B = -1$	A1	
	Form and solve equations for a or b	M1	e.g. $a = B - A + 2, \quad b = -B + 3$
	Obtain answer $a = -1$	A1	
	Obtain answer $b = 4$	A1	
		5	

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Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Use remainder theorem with $x = \frac{-1 \pm \sqrt{5}}{2}$	**M1**	Allow for correct use of either root in exact or decimal form.
	Obtain $-\frac{a}{2} \pm \frac{a\sqrt{5}}{2} + b = \frac{9}{2} \mp \frac{\sqrt{5}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations for a or b	**M1**	
	Obtain answer $a = -1$ from exact working	**A1**	
	Obtain answer $b = 4$ from exact working	**A1**	
		5	

Question	Answer	Marks	Guidance
4(i)	State $R = \sqrt{7}$	**B1**	
	Use correct trig formulae to find α	**M1**	e.g. $\tan\alpha = \frac{1}{\sqrt{6}}$, $\sin\alpha = \frac{1}{\sqrt{7}}$, or $\cos\alpha = \frac{\sqrt{6}}{\sqrt{7}}$
	Obtain $\alpha = 22.208°$	**A1**	ISW
		3	

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Question	Answer	Marks	Guidance
4(ii)	Evaluate $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$ to at least 1 d.p.	**B1FT**	$49.107°$ to 3 d.p. B1 can be implied by correct answer(s) later. The FT is on *their R*
			SC: allow B1 for a correct alternative equation e.g. $3\tan^2\theta - 2\sqrt{6}\tan\theta + 1 = 0$
	Use correct method to find a value of θ in the interval	**M1**	Must get to θ
	Obtain answer, e.g. $13.4°$	**A1**	Accept correct over-specified answers. 13.449…, 54.3425…
	Obtain second answer, e.g. $54.3°$ and no extras in the given interval	**A1**	Ignore answers outside the given interval.
		4	

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Question	Answer	Marks	Guidance
5	State $4xy + 2x^2 \dfrac{dy}{dx}$, or equivalent, as derivative of $2x^2 y$	B1	
	State $y^2 + 2xy \dfrac{dy}{dx}$, or equivalent, as derivative of xy^2	B1	
	Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)	*M1	$\dfrac{dy}{dx} = \dfrac{y^2 - 4xy}{2x^2 - 2xy}$
	Reject $y = 0$	B1	Allow from $y^2 - kxy = 0$
	Obtain $y = 4x$	A1	OE from correct numerator. ISW
	Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	DM1	$8x^3 - 16x^3 = a^3$ or $\dfrac{y^3}{8} - \dfrac{y^3}{4} = a^3$
	Obtain $y = -2a$	A1	With no errors seen
		7	

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Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	Rewrite as $y = \dfrac{a^3}{2x^2 - xy}$ and differentiate	**M1**	Correct use of function of a function and implicit differentiation
	Obtain correct derivative (in any form)	**A1**	$\dfrac{dy}{dx} = \dfrac{-a^3\left(4x - y - x\dfrac{dy}{dx}\right)}{\left(2x^2 - xy\right)^2}$
	set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)	***M1**	
	Obtain $4x - y = 0$	**A1**	
	Confirm $2x^2 - xy \neq 0$	**B1**	$x = 0$ and $2x = y$ both give $a = 0$
	Obtain an equation in y (or in x) and solve for y (or for x)	**DM1**	$8x^3 - 16x^3 = a^3$ or $\dfrac{y^3}{8} - \dfrac{y^3}{4} = a^3$
	Obtain $y = -2a$	**A1**	With no errors seen
		7	

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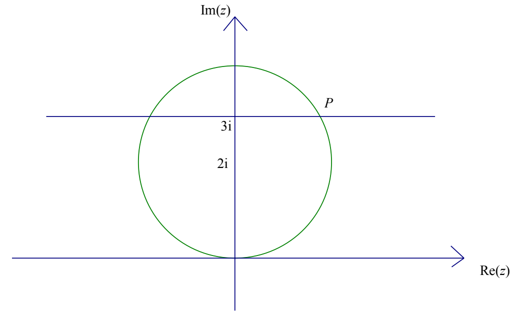
Question	Answer	Marks	Guidance
6	Separate variables correctly to obtain $\int \frac{1}{x+2}\,dx = \int \cot\frac{1}{2}\theta\ d\theta$	B1	Or equivalent integrands. Integral signs SOI
	Obtain term $\ln(x+2)$	B1	Modulus signs not needed.
	Obtain term of the form $k\ln\sin\frac{1}{2}\theta$	M1	
	Obtain term $2\ln\sin\frac{1}{2}\theta$	A1	
	Use $x = 1$, $\theta = \frac{1}{3}\pi$ to evaluate a constant, or as limits, in an expression containing $p\ln(x+2)$ and $q\ln\left(\sin\frac{1}{2}\theta\right)$	M1	Reach C = an expression or a decimal value
	Obtain correct solution in any form e.g. $\ln(x+2) = 2\ln\sin\frac{1}{2}\theta + \ln 12$	A1	ln12 = 2.4849….. Accept constant to at least 3 s.f. Accept with $\ln 3 - 2\ln\frac{1}{2}$
	Remove logarithms and use correct double angle formula	M1	Need correct algebraic process. $\left(\frac{x+2}{12} = \frac{1-\cos\theta}{2}\right)$
	Obtain answer $x = 4 - 6\cos\theta$	A1	
		8	

Question	Answer	Marks	Guidance
7(a)	Substitute and obtain a correct horizontal equation in x and y in any form	**B1**	$zz^* + \mathrm{i}z - 2z^* = 0 \Rightarrow$ $x^2 + y^2 + \mathrm{i}x - y - 2x + 2\mathrm{i}y = 0$ Allow if still includes brackets and/or i^2
	Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to zero OE	***M1**	For their horizontal equation
	Obtain two correct equations e.g. $x^2 + y^2 - y - 2x = 0$ and $x + 2y = 0$	**A1**	Allow $\mathrm{i}x + 2\mathrm{i}y = 0$
	Solve for x or for y	**DM1**	
	Obtain answer $\frac{6}{5} - \frac{3}{5}\mathrm{i}$ and no other	**A1**	OE, condone $\frac{1}{5}(6 - 3\mathrm{i})$
		5	

Question	Answer	Marks	Guidance
7(b)(i)	Show a circle with centre 2i and radius 2	**B1**	
	Show horizontal line $y = 3$ – in first and second quadrant	**B1**	
			SC: For clearly labelled axes not in the conventional directions, allow B1 for a fully 'correct' diagram.
		2	
7(b)(ii)	Carry out a complete method for finding the argument. (Not by measuring the sketch)	**M1**	$\left(z = \sqrt{3} + 3\text{i}\right)$ Must show working if using 1.7 in place of $\sqrt{3}$.
	Obtain answer $\frac{1}{3}\pi$ (or $60°$)	**A1**	**SC:** Allow B2 for 60° with no working
		2	

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Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 4$, $B = -1$, $C = 0$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
8(ii)	Integrate and obtain term $2\ln(2x-1)$	**B1FT**	The FT is on A. $\dfrac{1}{2}A\ln(2x-1)$
	Integrate and obtain term of the form $k\ln(x^2+2)$	***M1**	From $\dfrac{nx}{x^2+2}$
	Obtain term $-\dfrac{1}{2}\ln(x^2+2)$	**A1FT**	The FT is on B
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab\neq 0$	**DM1**	$2\ln 9(-2\ln 1)-\frac{1}{2}\ln 27+\frac{1}{2}\ln 3$
	Obtain answer ln 27 after full and correct exact working	**A1**	ISW
		5	

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Question	Answer	Marks	Guidance
9(i)	Commence integration by parts, reaching $ax\sin\frac{1}{3}x - b\int\sin\frac{1}{3}x\,dx$	*M1	
	Obtain $3x\sin\frac{1}{3}x - 3\int\sin\frac{1}{3}x\,dx$	A1	
	Complete integration and obtain $3x\sin\frac{1}{3}x + 9\cos\frac{1}{3}x$	A1	
	Substitute limits correctly and equate result to 3 in an integral of the form $px\sin\frac{1}{3}x + q\cos\frac{1}{3}x$	DM1	$3 = 3a\sin\frac{a}{3} + 9\cos\frac{a}{3}(-0) - 9$
	Obtain $a = \dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ correctly	A1	With sufficient evidence to show how they reach the given equation
		5	
9(ii)	Calculate values at a = 2.5 and a = 3 of a relevant expression or pair of expressions.	M1	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving f(a) = 0, f(2.5) = 0.179. and f(3) = –0.173 or if $f(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4 \Rightarrow f(2.5) = -0.13.., f(3) = 0.145...$
	Complete the argument correctly with correct calculated values	A1	Accept values to 1 sf. or better
		2	

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Question	Answer	Marks	Guidance
9(iii)	Use the iterative process $a_{n+1} = a_{n+1}\dfrac{4-3\cos\frac{1}{3}a_n}{\sin\frac{1}{3}a_n}$ correctly at least once	**M1**	
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	**A1**	
	Obtain final answer 2.736	**A1**	
		3	

Question	Answer	Marks	Guidance
10(i)	Express general point of l in component form e.g. $(1 + \lambda, 3 - 2\lambda, -2 + 3\lambda)$	**B1**	
	Substitute in equation of p and solve for λ	**M1**	
	Obtain final answer $\frac{5}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}$ from $\lambda = \frac{2}{3}$	**A1**	OE Accept $1.67\mathbf{i}+1.67\mathbf{j}$ or better
		3	

Question	Answer	Marks	Guidance
10(ii)	Use correct method to evaluate a scalar product of relevant vectors e.g. (**i** – 2**j** + 3**k**).(2**i** + **j** – 3**k**)	**M1**	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\lvert\sin\theta\rvert = \frac{9}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	
	Alternative method for question 10(ii)		
	Use correct method to evaluate a vector product of relevant vectors e.g. (**i** – 2**j** + 3**k**)x(2**i** + **j** – 3**k**)	**M1**	
	Using the correct process for calculating the moduli, divide the modulus of the vector product by the product of the moduli of the two vectors and evaluate the inverse sine or cosine of the result	**M1**	$\cos\theta = \frac{\sqrt{115}}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	

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Question	Answer	Marks	Guidance
10(iii)	State $a - 2b + 3c = 0$ or $2a + b - 3c = 0$	B1	
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	M1	Could use $2a + b - 3c = 0$ and $\begin{cases} a + 3b - 2c = d \\ \frac{5}{3}a + \frac{5}{3}b = d \end{cases}$ i.e. use two points on the line rather than the direction of the line. The second M1 is not scored until they solve for d.
	Obtain $a : b : c = 3 : 9 : 5$	A1	OE
	Substitute a, b, c and a relevant point in the plane equation and evaluate d	M1	Using their calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$	A1	OE
	Alternative method for question 10(iii)		
	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	M1	
	Obtain two correct components	A1	
	Obtain correct answer, e.g. $3\mathbf{i} + 9\mathbf{j} + 5\mathbf{k}$	A1	
	Use the product and a relevant point to find d	M1	Using *their* calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$, or equivalent	A1	OE

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Question	Answer	Marks	Guidance
10(iii)	**Alternative method for question 10(iii)**		
	Attempt to form a 2-parameter equation with relevant vectors	**M1**	
	State a correct equation e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**A1**	
	State 3 equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain answer $3x + 9y + 5z = 2$	**A1**	OE
		5	